\[ B^{*}_{s,d} \rightarrow \mu^+\mu^- \] and its impact on \[ B_{s,d} \rightarrow \mu^+\mu^- \]

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Outline

- Introduction

Based on arXiv: 1511.XXXXX
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- Calculation and numerical result
  - The amplitude of $B_{s,d}^{(*)} \rightarrow \mu^+ \mu^-$
  - The impact of $B_{s,d}^* \rightarrow B_{s,d} \rightarrow \mu^+ \mu^-$

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  - The parameters and numerical
- Summary

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Introduction
The leptonic decays of the $B_{s,d}$ mesons play an important role in the standard model (SM) and the new physics (NP)\cite{1, 2}.

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It are highly suppressed in the SM for the flavor changing neutral current decays are generated through W-box and Z-penguin diagrams.
The leptonic decays of the $B_{s,d}$ mesons play an important role in the standard model (SM) and the new physics (NP)\cite{1, 2}.

It are highly suppressed in the SM for the flavor changing neutral current decays are generated through W-box and Z-penguin diagrams.

However, the branching ratios of leptonic decays of scalar meson undergo an additional helicity suppression factor by $m_\mu^2/M_S^2$, where $m_\mu$ and $M_S$ denote masses of the muon lepton and the scalar meson, respectively.
Figure: **Feynman Diagram related with** $B_s \to \mu^+ \mu^-$ decay[2]:

- **a, b**, $\pi^+$, $B_u$, $B_s$ meson decay through $W$ in the SM;
- **c**, the forbidden $B_s$ decay through $Z$ in the SM;
- **d, e**, the SM allowed higher-order processes for the $B_s \to \mu^+ \mu^-$;
- **f, g**, examples of $B_s \to \mu^+ \mu^-$ in NP.
$B_{s,d} \rightarrow \mu^+ \mu^-$ measured by the CMS and LHCb

Figure: $B_{s,d} \rightarrow \mu^+ \mu^-$ measured by the CMS and LHCb, LHCb, CMS Collaboration Nature 522 (2015) 68–72 [2].
$B_{s,d} \rightarrow \mu^+ \mu^-$ measured by the CMS and LHCb

Table: $B_{s,d} \rightarrow \mu^+ \mu^-$ measured by the CMS and LHCb collaborations [2] and predicted within the SM [1] included NNLO QCD [3] and NLO EW [4] corrections.

<table>
<thead>
<tr>
<th></th>
<th>EX</th>
<th>SM</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Br[B_d \rightarrow \mu^+ \mu^-]$</td>
<td>$(3.9_{-1.4}^{+1.6})10^{-10}$</td>
<td>$(1.06 \pm 0.09)10^{-10}$</td>
<td>$2.2\sigma$</td>
</tr>
<tr>
<td>$Br[B_s \rightarrow \mu^+ \mu^-]$</td>
<td>$(2.8_{-0.6}^{+0.7})10^{-9}$</td>
<td>$(3.66 \pm 0.23)10^{-9}$</td>
<td>$1.2\sigma$</td>
</tr>
<tr>
<td>$Br[B_d \rightarrow \mu^+ \mu^-] / Br[B_s \rightarrow \mu^+ \mu^-]$</td>
<td>$0.14_{-0.06}^{+0.08}$</td>
<td>$0.0295_{-0.0025}^{+0.0028}$</td>
<td>$2.3\sigma$</td>
</tr>
</tbody>
</table>

New Physics?
Experiment VS SM: $B_{s,d} \rightarrow \mu^+ \mu^-$


Figure: Experiment VS SM: $B_{s,d} \rightarrow \mu^+ \mu^-$ [2]
New physics @ $B_{s,d} \rightarrow \mu^+ \mu^-$

1. Two-Higgs doublet models [5]

The minimal supersymmetric standard model (MSSM) [6].
The next minimal supersymmetric standard model (NMSSM) [8].
The dark matter [10].
And so on [11] ...
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4. $e^+ e^- \to B_{s,d}^*$ are considered too in Ref. [12].
Calculation
The effective Lagrangian related with $b\bar{s} \rightarrow \mu^+\mu^-$ is\cite{13, 14}

$$L = \mathcal{N}\left[ C_7(\mu_b)O_7^\gamma + C_9(\mu_b)O_9^V + C_{10}(\mu_b)O_{10}^A \right], \quad (1)$$

where $\mathcal{N} = \frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^* \frac{e^2}{4\pi^2}$. The superscript $\gamma$, $V$, and $A$ denote the contributions from photon, vector current, and axial vector current respectively.
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2. The local operators $O_{7,9,10}$ read as

$$O_7^\gamma = - \frac{2im_b}{q^2} q_\nu (\bar{s}\sigma_{\mu\nu} P_R b)(\bar{\mu}\gamma^\mu \mu), \quad (2)$$

$$O_9^V = (\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu \mu), \quad (3)$$

$$O_{10}^A = (\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu \gamma_5 \mu), \quad (4)$$

where $P_L = (1 - \gamma_5)/2$, $P_R = (1 + \gamma_5)/2$, $q = p_\mu + p_{\bar{\mu}}$. 
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where $P_L = (1 - \gamma_5)/2$, $P_R = (1 + \gamma_5)/2$, $q = p_\mu + p_\bar{\mu}$.

The Wilson coefficients are $C_{7,9,10}(\mu_b) = (-0.29, 4.07, -4.31)$ at $\mu_b = 4.8$ GeV.
The relation between the quark and the mesons

The relation between the quark level operators and the mesons are described as

\[
\langle 0 | \bar{s} \gamma^\mu b | B_s^*(q, \varepsilon) \rangle = m_{B_s^*} f_{B_s^*} \varepsilon^\mu, \tag{5}
\]

\[
\langle 0 | \bar{s} \sigma^{\mu\nu} b | B_s^*(q, \varepsilon) \rangle = -i f_{B_s^*} (q^\mu \varepsilon^\nu - \varepsilon^\mu q^\nu), \tag{6}
\]

\[
\langle 0 | \bar{s} \gamma^\mu \gamma^5 b | B_s(q) \rangle = -i f_{B_s} q^\mu, \tag{7}
\]

where the three decay constants \( f_{B_s}, f_{B_s^*}, f_{B_s^*} \) are related in the heavy-quark limit [15],

\[
f_{B_s^*} = f_{B_s^*} = f_{B_s}. \tag{8}
\]
Amplitudes of $B_s^*(B_s) \rightarrow \mu^+\mu^-$

Then the amplitudes of $B_s^*(B_s) \rightarrow \mu^+\mu^-$

\[ \mathcal{M}(B_s^* \rightarrow \mu^+\mu^-) = \frac{\mathcal{N}}{2} \frac{m_{B_s^*}}{m_B^*} \bar{\mu} \ell [C_{V}^{\text{eff}} + C_{10} \gamma_5] \mu, \]

\[ \mathcal{M}(B_s \rightarrow \mu^+\mu^-) = i f_{B_s} \mathcal{N} C_{10} m_\mu \bar{\mu} \gamma_5 \mu, \]  \hspace{1cm} (9)

where

\[ C_{V}^{\text{eff}} = C_9 + 2 m_b / m_{B_s^*} C_7. \]  \hspace{1cm} (10)

The helicity suppression factor $m_\mu^2/m_M^2$ in the decay width is removed in the vector meson decay.
Width of $B_s^*(B_s) \to \mu^+ \mu^-$

The decay widths of $B_s^*(B_s) \to \mu^+ \mu^-$

\[
\Gamma(B_s^* \to \mu^+ \mu^-) = \frac{G_f^2 \alpha_{em}^2}{96\pi^3} |V_{tb}V_{ts}^*|^2 \left( |C_{10}|^2 + \left| C_{V}^{\text{eff}} \right|^2 \right) m_{B_s^*}^3 f_{B_s^*}^2 \\
\times \left( 1 + \mathcal{O}(m_\mu^2/m_{B_s}^2) \right)
\]

\[
\Gamma(B_s \to \mu^+ \mu^-) = \frac{G_f^2 \alpha_{em}^2}{16\pi^3} |V_{tb}V_{ts}^*|^2 |C_{10}|^2 m_\mu^2 m_{B_s} f_{B_s}^2 \\
\times \left( 1 + \mathcal{O}(m_\mu^2/m_{B_s}^2) \right)
\]

\[
\frac{\Gamma(B_s^* \to \mu^+ \mu^-)}{\Gamma(B_s \to \mu^+ \mu^-)} = \frac{1}{6} \frac{|C_{10}|^2 + \left| C_{V}^{\text{eff}} \right|^2}{|C_{10}|^2} \frac{m_{B_s^*}^3 f_{B_s^*}^2}{m_\mu^2 m_{B_s} f_{B_s}^2} \\
\times \left( 1 + \mathcal{O}(m_\mu^2/m_{B_s}^2) \right)
\]

(11)
The $B_{s,d}^*$ will impact on the leptonic decay of $B_{s,d}$ through $B_{s,d} \to B_{s,d}^* \gamma^* \to \mu^+ \mu^-$. 

Figure: Feynman diagrams of $B_{s,d} \to B_{s,d}^* \gamma^* \to \mu^+ \mu^-$. 
The vertex of $B_s B_s^* \gamma$

The vertex of $B_{s,d} \rightarrow B_{s,d}^* \gamma$ is given as

$$M_{B_s B_s^* \gamma} = \frac{g_{B_s B_s^* \gamma}}{m_{b_s^*}} \exp[i\phi] \epsilon^{\mu \nu \alpha \beta}_{\gamma} \epsilon_{\gamma p_{\gamma}^{\nu} \epsilon_{B_s^* p_{B_s^*}^{\beta}},}$$

(12)

Where $g_{B_s B_s^* \gamma}$ is the positive dimensionless vector-scalar-photon coupling constant and $\phi$ is the real hadronic phase angular.
There are logarithmical diverges in the evaluation of loop integrals. Thus, a form factor is introduced in the $B_s B_s^* \gamma$ or $B_s^* \mu^+ \mu^-$ vertex [16]:

$$ F(n) = \left( \frac{\Lambda^2 - m^2_n}{\Lambda^2 - p^2_n} \right), $$

(13)

where $n = \gamma, B_s^*$, or $\mu$, and the large energy cut $\Lambda$ is also corresponded to Pauli-Villars regularization

$$ \frac{1}{p_n^2 - m_n^2} F(n) = \frac{1}{p_n^2 - m_n^2} - \frac{1}{p_n^2 - \Lambda^2} . $$

(14)
The amplitude from $B_s \rightarrow B_{s}^{*}\gamma^{*} \rightarrow \mu^{+}\mu^{-}$

The amplitude from $B_s \rightarrow B_{s}^{*}\gamma^{*} \rightarrow \mu^{+}\mu^{-}$ can be written as

$$
\mathcal{M}(B_s \rightarrow B_{s}^{*}\gamma^{*} \rightarrow \mu^{+}\mu^{-}) = e g_{B_s B_{s}} \exp[i \phi_s] R_n(\Lambda) \times C_{V_{\alpha}}^{eff} f_{B_{s}^{*}} m_{\mu} \bar{\mu} \gamma_5 \mu. \quad (15)
$$

$m_{\mu}$ reappear in the amplitude of leptonic decay of scalar meson.

There is a term $\log \frac{m_{\mu}^2}{m_{B_{s}^{*}}^2}$ in $R_n$ when the virtual photon and $\mu$ lepton are collinear. Compared with Eq.(9), the amplitude will be add a term

$$
F(B_{s}^{*}) = \frac{\mathcal{M}(B_s \rightarrow B_{s}^{*}\gamma^{*} \rightarrow \mu^{+}\mu^{-})}{\mathcal{M}(B_s \rightarrow \mu^{+}\mu^{-})} = C_{V_{\alpha}}^{eff} f_{B_{s}^{*}} eg_{B_s B_{s}} \exp[i \phi] R_n(\Lambda) \quad (16)
$$
Figure: $R_{B^*_s}$, $R_\gamma$, $R_\mu$, and $\bar{R} = 1/3(R_{B^*_s} + R_\gamma + R_\mu)$ of $B_s \to \mu^+\mu^-$ defined in Eq.(15) as a function the high energy cut.
$g B_s B^*_s \gamma$ can be estimated through

1. the heavy-quark and chiral effective theories [17, 18, 7, 9],...
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2. the light cone QCD sum rules[19]
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1. the heavy-quark and chiral effective theories [17, 18, 7, 9],
2. the light cone QCD sum rules[19]
3. the radiative M1 decay widths of $B_s^* \rightarrow B_s \gamma$[20, 21].
$g_{B_s B_s^* \gamma}$ from $D^{*+} \rightarrow D^+ \gamma$

In the heavy quark and chiral effective theories,

$$g_{B_s B_s^* \gamma} = e m_{B_s^*} \left( \frac{1}{3 m_b} + \frac{\beta}{3} - \frac{g^2 m_K}{4 \pi f_K^2} \right).$$

$$g_{B_d B_d^* \gamma} = e m_{B_d^*} \left( \frac{1}{3 m_b} + \frac{\beta}{3} - \frac{g^2 m_{\pi}}{4 \pi f_{\pi}^2} \right). \quad (17)$$

And $\beta, g$ are determined from $D^{*+}$ decay

$$\Gamma(D^{*+} \rightarrow D^+ \gamma) = \frac{\alpha_{EM}}{3} \left( \frac{2}{3 m_c} - \frac{1}{3} \beta + \frac{g^2 m_{\pi}}{4 \pi f_{\pi}^2} \right)^2 |\vec{k}|^3,$$

$$\Gamma(D^{*+} \rightarrow D^0 \pi^+) = \frac{g^2}{6 \pi f_{\pi}^2} |\vec{p_\pi}|^3. \quad (18)$$
$g_{B_s B_s^* \gamma}$ from $D^{*+} \rightarrow D^+ \gamma$

$\mathcal{B}(D^{*+} \rightarrow D^+ \gamma) = (1.6 \pm 0.4)\%$, $\mathcal{B}(D^{*+} \rightarrow D^0 \pi^+) = (67.7 \pm 0.5)\%$, and $\Gamma_{D^{*+}} = (83.3 \pm 1.8)$ keV, [22], we can get $\beta, g$ in charm meson

$$g = 0.55 \pm 0.02$$
$$\beta = (3.5 \pm 0.2) \text{ GeV}^{-1},$$

(19)

Thus the the heavy quark and chiral effective theories give

$$g_{B_s B_s^* \gamma} = 1.2 \pm 0.2$$
$$g_{B_d B_d^* \gamma} = 1.7 \pm 0.2.$$ 

(20)
The calculated M1 transition rates for the strange bottom mesons are listed in Table 12.

Table 11: The M1 decay widths of the $B_0$ ($b\bar{d}$) mesons

<table>
<thead>
<tr>
<th>Transition</th>
<th>NRIA</th>
<th>RIA</th>
<th>RIA + cf.</th>
<th>RIA + cf. + ex.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^{*0}_s \to B^0$</td>
<td>152 eV</td>
<td>51.9 eV</td>
<td>25.1 eV</td>
<td>9.55 eV</td>
</tr>
<tr>
<td>$B^{*0}_s' \to B^0$</td>
<td>149 meV</td>
<td>5.39 keV</td>
<td>9.73 keV</td>
<td>12.2 keV</td>
</tr>
<tr>
<td>$B^{*0}_s' \to B^0'$</td>
<td>27.7 eV</td>
<td>9.59 eV</td>
<td>48.3 eV</td>
<td>40.6 eV</td>
</tr>
<tr>
<td>$B^0' \to B^{*0}_s$</td>
<td>34.6 eV</td>
<td>4.31 keV</td>
<td>7.95 keV</td>
<td>9.72 keV</td>
</tr>
</tbody>
</table>

Table 13: The M1 decay widths of the $B_s$ ($b\bar{s}$) mesons

<table>
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<tr>
<th>Transition</th>
<th>NRIA</th>
<th>RIA</th>
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<th>RIA + cf. + ex.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^*_s \to B_s$</td>
<td>116 eV</td>
<td>46.4 eV</td>
<td><strong>945 meV</strong></td>
<td><strong>148 meV</strong></td>
</tr>
<tr>
<td>$B^*_s' \to B_s$</td>
<td>2.39 eV</td>
<td>3.87 keV</td>
<td>2.29 keV</td>
<td>3.22 keV</td>
</tr>
<tr>
<td>$B^*_s' \to B'_s$</td>
<td>20.9 eV</td>
<td>8.26 eV</td>
<td>12.4 eV</td>
<td>9.76 eV</td>
</tr>
<tr>
<td>$B'_s \to B^*_s$</td>
<td>10.6 eV</td>
<td>3.12 keV</td>
<td>1.84 keV</td>
<td>2.50 keV</td>
</tr>
</tbody>
</table>

Figure: M1 transition widths of $B^*_{s,d}$ in Hep-ph/9908485[20].
$B_s^* \rightarrow \mu^+\mu^-$ in CMS and LHCb measurement?

If $\Gamma_{\text{tot}}[B_s^*] = \Gamma[B_s^* \rightarrow B_s\gamma] = 0.945$ eV, then we can get

$$\frac{Br[B_s^* \rightarrow \mu^+\mu^-]}{Br[B_s \rightarrow \mu^+\mu^-]} = \frac{\Gamma_{\text{tot}}[B_s]}{\Gamma_{\text{tot}}[B_s^*]} \left(\frac{|C_{10}|^2 + |C_{\text{eff}}^V|^2}{6|C_{10}|^2}\right) \frac{m_{B_s^*}^3 f_{B_s^*}^2}{m_{\mu}^2 m_{B_s} f_{B_s}^2} = 0.34$$

Considered $\sigma[B_s^*] \sim 3/4\sigma[B_s]$, then the CMS and LHCb measurement $Br[B_s \rightarrow \mu^+\mu^-] = 2.9 \times 10^{-9}$ would be

$$Br[B_s^* \rightarrow \mu^+\mu^-] = 0.8 \times 10^{-9}$$
$$Br[B_s \rightarrow \mu^+\mu^-] = 2.3 \times 10^{-9}$$

(22)

For $Br[B_{s,d}^* \rightarrow e^+e^-] \sim Br[B_s^* \rightarrow \mu^+\mu^-]$, more information about $e^+e^-$ distribution of CMS and LHCb measurement is useful.

Unfortunately, in most calculation,

$$\Gamma[B_d^* \rightarrow B_d\gamma] = (200 \sim 400) \text{ eV}$$
$$\Gamma[B_s^* \rightarrow B_s\gamma] = (50 \sim 150) \text{ eV}$$

(23)
Figure: $B_{s,d} \rightarrow \mu^+ \mu^-$ measured by the CMS and LHCb [2].
$g_{B_s B_s^* \gamma}$ in the light cone QCD sum rules[19]

$$g_{B_d B_d^* \gamma} = 2.3 \pm 0.3. \quad (24)$$

In the numerical calculation, the parameters are selected as

$$g_{B_d B_d^* \gamma} = 2.0 \pm 0.5,$$
$$g_{B_s B_s^* \gamma} = 1.2 \pm 0.4,$$
$$\Lambda = (3 \pm 1)m_{B_s^*}, \quad (25)$$
The missing amplitude

The contributions of $B_{s,d}^*$ will modify the amplitude by a factor

$$F(B_d^*) = -\exp[i\phi_d](36^{+4}_{-3} \pm 9) \times 10^{-3},$$
$$F(B_s^*) = -\exp[i\phi_s](22 \pm 2 \pm 7) \times 10^{-3}. \tag{26}$$

The error bars are from the $\Lambda$ and vector-scalar-photon coupling constant respectively.
For the missing information about the hadronic phase angular $\phi_d$ and $\phi_s$, the impact of $B^*_s,d$ on $B_{s,d}$ leptonic decay widths are estimated as a new error bar,

\[
Br[B_d \to \mu^+ \mu^-] = (10.6 \pm 0.9 \pm 0.8)10^{-11},
\]
\[
Br[B_s \to \mu^+ \mu^-] = (36.6 \pm 2.3 \pm 1.6)10^{-10}.
\] (27)
Summary
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As a summary, the impact of $B_{s,d}^* \rightarrow \mu^+ \mu^-$ on $B_{s,d} \rightarrow \mu^+ \mu^-$ is studied here.

1. We find that the amplitude is modified by a factor $3.6\%$ and $2.2\%$ for $B_d$ and $B_s$ respectively.

4. We need more information about $B_{s,d}^* \rightarrow e^+ e^-$, and $B_{s,d}^* \rightarrow B_{s,d} \gamma$. BSE, Lattice, experimental and so on.
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2. For missing information about the phase angular, we only estimate the error bar of branch ratio is enlarged by an additional term of 7.2% and 4.4% for $B_d$ and $B_s$ respectively within the standard model prediction.
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2. For missing information about the phase angular, we only estimate the error bar of branch ratio is enlarged by an additional term of 7.2% and 4.4% for $B_d$ and $B_s$ respectively within the standard model prediction.

3. Then the new standard model predictions are
   $Br[B_d \rightarrow \mu^+ \mu^-] = (10.6 \pm 0.9 \pm 0.8)10^{-11}$ and
   $Br[B_s \rightarrow \mu^+ \mu^-] = (36.6 \pm 2.3 \pm 1.6)10^{-10}$.

4. We need more information about $B_{s,d}^* \rightarrow e^+ e^-$, and $B_{s,d}^* \rightarrow B_{s,d}\gamma$. BSE, Lattice, experimental and so on.
Thanks!


arXiv:1510.06851.


