Scaling behavior of charged hadron $p_T$ distributions in $pp$ and $p\bar{p}$ collisions

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One of the main goals in high energy collisions is to investigate the dynamics for particle productions.

Several approaches are utilized to search for regularities in the particle productions.

In the high $p_T$ region, the particle production is governed by hard physics $\rightarrow$ perturbative QCD.

In the low $p_T$ region, the particle production is governed by soft physics $\rightarrow$ non-perturbative theory or model.

One novel approach is to search for a scaling behavior of the particle productions that is valid both in low and high $p_T$ regions.
Bjorken scaling in DIS: the parton distribution functions \( f(x) \) only depend on \( x \) and are independent of \( Q^2 \).

KNO scaling: multiplicity distributions \( P_n \) are different for different energy scales. However, the multiplicity distributions exhibit a scaling behavior when they are presented as a function of \( z = n/\langle n \rangle \): 

\[
\Psi(z) = \langle n \rangle P_n
\]
A scaling behavior of the pion $p_T$ spectra with different centralities at midrapidity was found in Au+Au collisions.

$$\Phi(z) = A(N)K^2(N)\frac{1}{2\pi p_T} \frac{d^2N_\pi}{d\eta dp_T}$$, $z = p_T/K(N)$

$A$ and $K$ depend on the number of participants, $N_{\text{part}}$, in the collisions. $N_{\text{part}}$ is utilized to quantify the centrality of collisions.

This scaling behavior was extended to non-central rapidity region in \( Au+Au \) and \( d+Au \) collisions.

\[
\Psi(u) = \langle z \rangle^2 \Phi(\langle z \rangle u) / \int_0^\infty \Phi(z)zdz, \quad u = \frac{z}{\langle z \rangle} = \frac{p_T}{\langle p_T \rangle}
\]

Scaling behavior in the $p_T$ spectra of $p$ and $\bar{p}$ produced in Au+Au collisions with different centralities.

Is there a similar scaling behavior in the proton-proton ($pp$) or proton-antiproton ($p\bar{p}$) collisions? Yes, there is.
The $p_T$ spectra of primary charged hadrons produced in the $pp$ ($p\bar{p}$) collisions with $\sqrt{s} =$ 0.9, 2.36 and 7 (0.63, 1.8 and 1.96) TeV at the CMS (CDF) detector.

The primary charged hadrons refer to the charged hadrons produced in the inelastic non-single-diffractive (NSD) interactions.
Method to search for the scaling behavior

- \( z = \frac{p_T}{K}, \Phi(z) = A \cdot (2\pi p_T)^{-1} d^2 N/dp_T dy \big|_{p_T=Kz} \)
- \( K \) and \( A \) are free parameters
- By choosing proper \( K \) and \( A \), the scaled \( p_T \) spectra, \( \Phi(z) \), at different energy scales can be put into one curve.
- As a convention, \( K \) and \( A \) are set to be 1 for the highest energy collisions.
- It is obvious that with different choices of \( A \) and \( K \) for the highest energy collisions, we get different scaling functions.
- In order to get rid of this arbitrary, we define \( u = \frac{z}{\langle z \rangle} = \frac{p_T}{\langle p_T \rangle} \)
  and \( \Psi(u) = \langle z \rangle^2 \Phi(\langle z \rangle u) / \int_0^\infty \Phi(z)zdz \).
- \( \langle z \rangle = \int_0^\infty z\Phi(z)zdz / \int_0^\infty \Phi(z)zdz, \int_0^\infty \Psi(u)udu = \int_0^\infty u\Psi(u)udu = 1 \)
Scaling behavior in $pp$ and $p\bar{p}$ collisions

- $\Phi_{pp}(z) = 27.63\exp(-6.93\nu + 0.44\nu^2 - 0.26\nu^3)$, $\nu = \ln(1 + z)$
- $\Phi_{p\bar{p}}(z) = 1085.12\exp(-6.56\nu - 0.64\nu^2 + 0.12\nu^3)$, $\nu = \ln(1 + z)$
- $R = \text{experimental data/fitted results}$

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<thead>
<tr>
<th>$\sqrt{s}$ (TeV)</th>
<th>$K$</th>
<th>$A$</th>
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<td>0.9</td>
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<td>0.85</td>
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<tr>
<td>2.36</td>
<td>0.88</td>
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<td>1.96</td>
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Scaling behavior in \( pp \) and \( p\bar{p} \) collisions

- \( u = z / \langle z \rangle, \langle z \rangle_{pp} = 0.53, \langle z \rangle_{p\bar{p}} = 0.48. \)
- \( \Psi_{pp}(u) = 7.81 \exp(-3.73v - 0.61v^2 - 0.02v^3), \ v = \ln(1 + u). \)
- \( \Psi_{p\bar{p}}(u) = 5.95 \exp(-2.75v - 1.44v^2 + 0.16v^3), \ v = \ln(1 + u). \)
Scaling behavior in $pp$ and $p\bar{p}$ collisions

- The scaling behavior could be validated experimentally.
- The ratio between the moments of the momentum distributions:
  \[
  \frac{\langle p_T^n \rangle}{\langle p_T \rangle^n} = \int_0^\infty u^n \Psi(u) u du, \quad n = 2, 3, 4, \ldots
  \]
- As an example, we calculate $\frac{\langle p_T^2 \rangle}{\langle p_T \rangle^2}$ using the measured data points in the $pp$ collisions with $\sqrt{s} = 0.9, 2.36$ and 7 TeV:
  
  $\frac{\langle p_T^2 \rangle}{\langle p_T \rangle^2} \bigg|_{\sqrt{s}=0.9, 2.36, 7 \text{ TeV}} = 1.68, 1.73, 1.85$.

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<tr>
<th>$n$</th>
<th>$pp$ collisions</th>
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<tr>
<td>2</td>
<td>1.93</td>
<td>1.90</td>
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<tr>
<td>3</td>
<td>6.23</td>
<td>6.39</td>
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<tr>
<td>4</td>
<td>29.93</td>
<td>42.04</td>
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<tr>
<td>5</td>
<td>188.92</td>
<td>663.58</td>
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The statistics origin of the scaling behavior

- $\Phi(z)$ is in an exponential form of $v$, rather than $z$.
- The statistical mechanics of the particle production in the $pp$ and $p\bar{p}$ collisions is thought to be non-extensive.
- The Tsallis entropy, $S_q(z) = (1 - \int p^q(z)dz)/(q - 1)$, is a non-extensive entropy.
- The Tsallis distribution is derived by maximizing the Tsallis entropy:
  \[
  \Phi(z) = C_q \left[ 1 - (1 - q) \frac{z}{z_0} \right]^{\frac{1}{1-q}}, \quad C_q, \ z_0 \text{ and } q \text{ are free parameters.}
  \]
- $|1 - q|$ is a measure of the non-extensivity. $q \to 1$, the Tsallis distribution becomes the BG distribution $\Phi(z) = C_1 \exp(-\frac{z}{z_0})$.
Statistics origin of the scaling behavior

- For the $pp$ collisions, $C_q = 22.04$, $z_0 = 0.17$ and $q = 1.127$.
- For the $p\bar{p}$ collisions, $C_q = 1131.52$, $z_0 = 0.15$ and $q = 1.125$.

Statistics origin of the scaling behavior

- \( \Psi_{Tsa.}(u) = 6.75 \left[ 1 - (1 - 1.127) \frac{u}{0.311} \right]^{\frac{1}{1-1.127}} \)
- \( \Psi_{Tsa.}(u) = 6.72 \left[ 1 - (1 - 1.125) \frac{u}{0.313} \right]^{\frac{1}{1-1.125}} \)

The agreement confirms the charged hadron system produced in the collisions is a non-extensive thermodynamics system.

- The non-extensivity of the system is described by the parameter \( q \), which is 1.127 (1.125) for the \( pp \) (\( p\bar{p} \)) collisions.

In order to understand the particle production mechanism behind this scaling behavior, the **model of percolation of strings** is utilized.

- Color strings are stretched between the two colliding hadrons. These strings will split into new strings with the emission of $q\bar{q}$ pairs.
- The average $p_T^2$ of the cluster with $n$ strings: $\langle p_T^2 \rangle_n = \sqrt{nS_1 / S_n} \langle p_T^2 \rangle_1$
- The $p_T$ distribution at energy scale $\sqrt{s'}$ could be related to the $p_T$ distribution at energy scale $\sqrt{s}$ by a linear transformation on $p_T$ at energy scale $\sqrt{s'}$: $p_T|_{\sqrt{s}} \to p_T|_{\sqrt{s'}} / ((nS_1 / S_n) \sqrt{s'} / (nS_1 / S_n) \sqrt{s})^{1/4}$.

maximum overlap, $S_n = S_1$, $nS_1 / S_n = n$

get in touch, $S_n = nS_1$, $nS_1 / S_n = 1$
Comparing the $p_T$ transformation in this model with the one used in the way to search for the scaling behavior, $p_T \rightarrow p_T/K$, we know that $K$ gives the ratio between the degrees of string overlap for the collisions at $\sqrt{s'}$ and $\sqrt{s}$.

For the $pp$ ($p\bar{p}$) collisions at CMS (CDF), $\sqrt{s}$ is set to be 7 (1.96) TeV and $\sqrt{s'}$ is set to be 0.9, 2.36 and 7 (0.63, 1.8 and 1.96) TeV.

Since the degree of string overlap, $nS_1/S_n$, grows with the increase of the energy scale.

$K$ should also grow with the increase of the energy scale.

That’s indeed what we observed.

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Conclusions

- There is a scaling behavior in the $p_T$ distributions of charged hadrons produced in the $pp$ or $p\bar{p}$ collisions with different energy scales.
- This scaling behavior is observed when a linear transformation is applied on $p_T$.
- The exponential and Tsallis distributions can both be applied to describe this scaling behavior within an accuracy of 20%.
- The particle systems produced in the $pp$ and $p\bar{p}$ collisions are non-extensive thermodynamics systems.
- The scaling behavior of the charged hadron $p_T$ distributions at different energy scales is successfully explained by the model of percolation of strings.
- Thanks for your attention.